

Inference rules in DL and SWRL

G. Falquet
C. Métral

Expressivity of DL

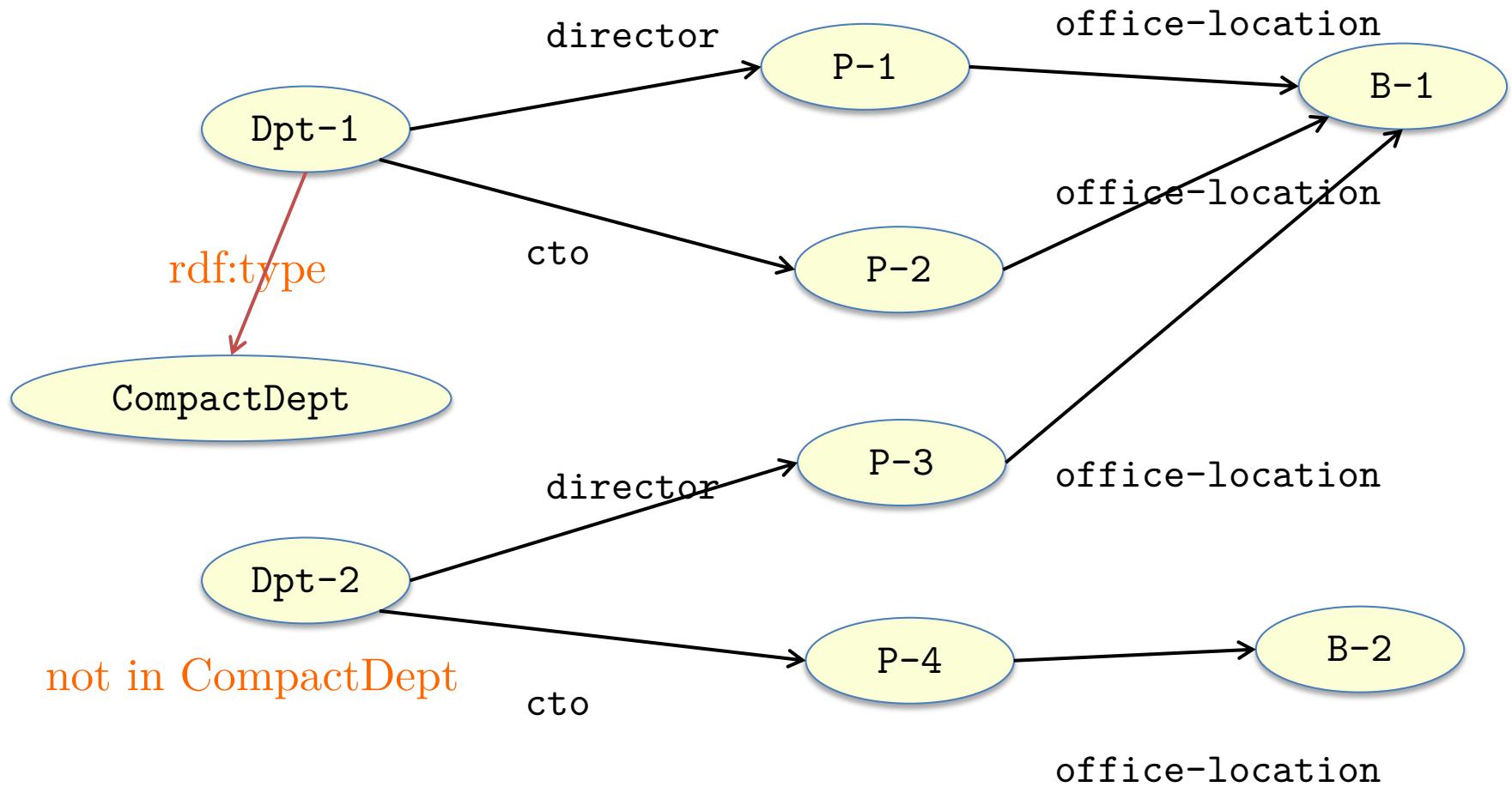
Vocabulary:

classes: Department, Employee, Building

properties: director, cto, office-location

How to define a class CompactDept to represent
departments that have their director and chief technology officer offices
located in the same building

CompactDept \equiv ???



In DL (OWL 2)

Impossible to define CompactDept in OWL-2

Many other examples cannot be defined in OWL-2

Theoretical reason: most DLs enjoy the [Tree Model Property](#).

if a Tbox has a model

then it has a model that doesn't contain cycles

A fact is a consequence of a Tbox if it is true in [every](#) model of the Tbox

⇒ no "cyclic fact" is a consequence of a TBox.

⇒ Need for another language to express these facts

Inference rules

Rules to produce

New **type** assertions

x is a member of class C

New **property** assertions

x is connected to y through property p

Inference rules

To produce **type** assertions

$$B_1 \wedge B_2 \wedge \cdots \wedge B_k \rightarrow C(x)$$

To produce new **property** assertions

$$B_1 \wedge B_2 \wedge \cdots \wedge B_k \rightarrow p(x, y)$$

B_i is either a class assertion $C(t)$ or a property assertion $p(u, v)$
 t, u, v are either individual names or variables

$$\text{Restaurant}(x) \wedge \text{hasMenu}(x, m) \wedge \text{contains}(m, \text{caviar}) \rightarrow \text{Expensive}(x)$$

$$\text{hasChild}(x, y) \rightarrow \text{hasParent}(y, x)$$

SWRL Rules - syntax

rule ::= antecedant \rightarrow consequent

antecedant ::= atom, atom, ...

consequent ::= atom, atom, ...

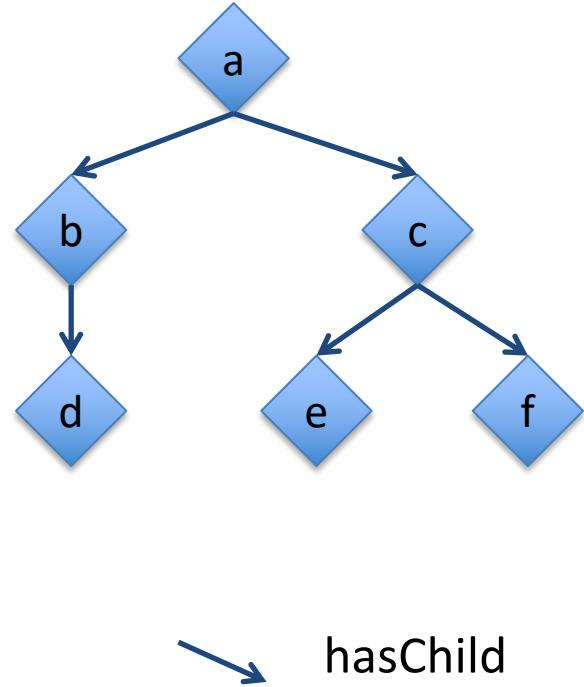
atom ::= description '(' i-object ')'
| dataRange '('" d-object "')'
| individualvaluedPropertyID '('" i-object i-object "')'
| datavaluedPropertyID '('" i-object d-object "')'
| sameAs '('" i-object i-object "')'
| differentFrom '('" i-object i-object "')'
| builtIn '('" builtinID { d-object } "')'

Person(?x), Person(?y), Person(?z), hasChild(?x, ?y), hasChild(?y, ?z) \rightarrow
hasGrandChild(?x, ?z)

Interpretation

- Find all the variable bindings that satisfy the antecedent
- For each such binding the consequent must be satisfied

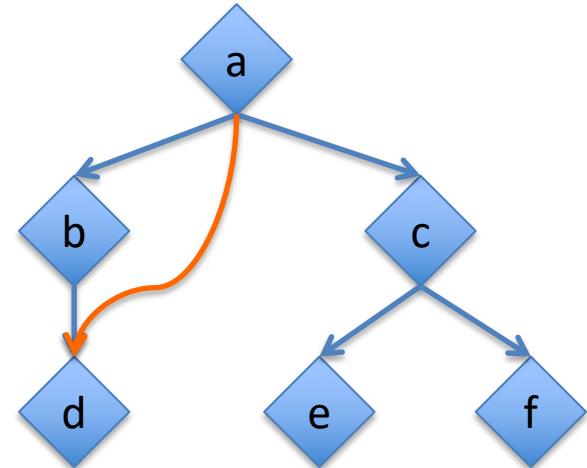
`hasChild(?x, ?y), hasChild(?y, ?z)`
→ `hasGrandChild(?x, ?z)`



Interpretation

`hasChild(?x, ?y), hasChild(?y, ?z)`
-> `hasGrandChild(?x, ?z)`

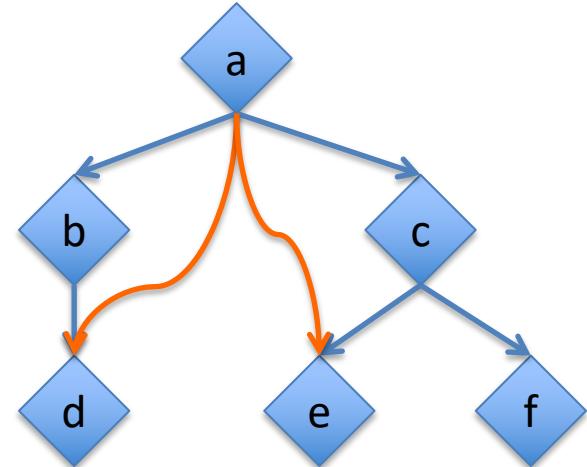
`x=a, y=b, z=d` -> `hasGrandChild(a,d)`



Interpretation

`hasChild(?x, ?y), hasChild(?y, ?z)`
 \rightarrow `hasGrandChild(?x, ?z)`

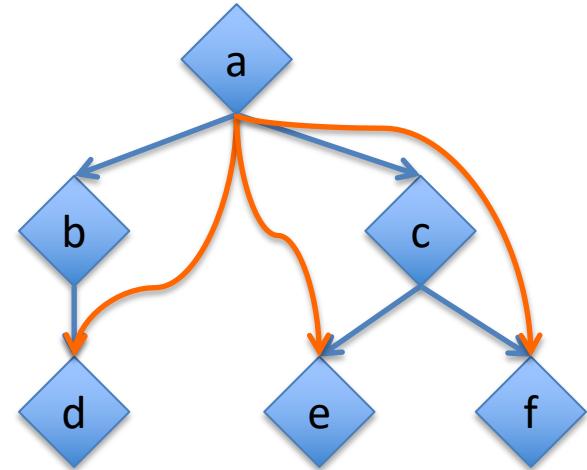
`x=a, y=b, z=d \rightarrow hasGrandChild(a,d)`
`x=a, y=c, z=e \rightarrow hasGrandChild(a,e)`



Interpretation

```
hasChild(?x, ?y), hasChild(?y, ?z)  
-> hasGrandChild(?x, ?z)
```

x=a, y=b, z=d → hasGrandChild(a,d)
x=a, y=c, z=e → hasGrandChild(a,e)
x=a, y=c, z=f → hasGrandChild(a,f)



Interpretation

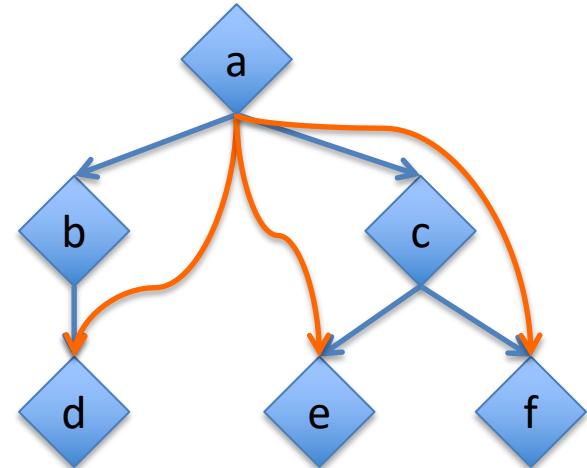
`hasChild(?x, ?y), hasChild(?y, ?z)`

\rightarrow `hasGrandChild(?x, ?z)`

`x=a, y=b, z=d` \rightarrow `hasGrandChild(a,d)`

`x=a, y=c, z=e` \rightarrow `hasGrandChild(a,e)`

`x=a, y=c, z=f` \rightarrow `hasGrandChild(a,f)`



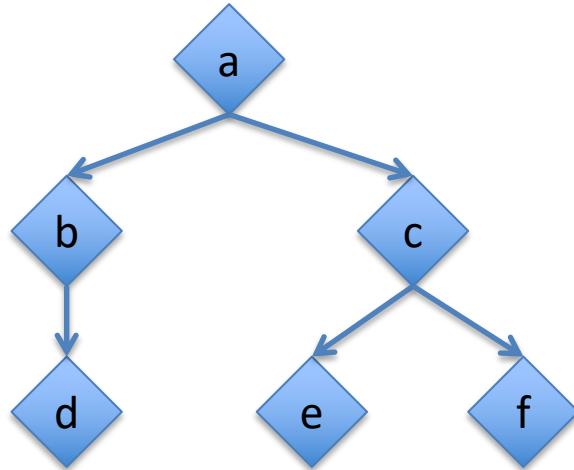
an interpretation that
satisfies the rule

DifferentFrom

Variables with different names may represent the same individual !

hasChild(?x, ?y), hasChild(?x, ?z)

-> **hasSibling**(?y, ?z)

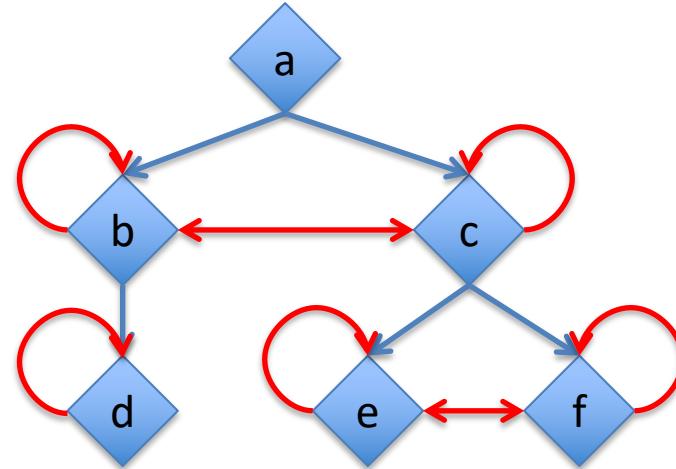


DifferentFrom

Variables with different names may represent the same individual !

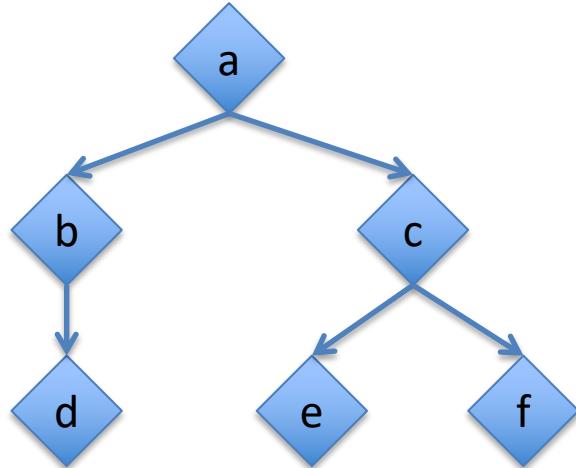
`hasChild(?x, ?y), hasChild(?x, ?z)`

\rightarrow `hasSibling(?y, ?z)`



DifferentFrom

```
hasChild(?x, ?y), hasChild(?x, ?z), DifferentFrom (?y, ?z)  
-> hasSibling(?y, ?z)
```

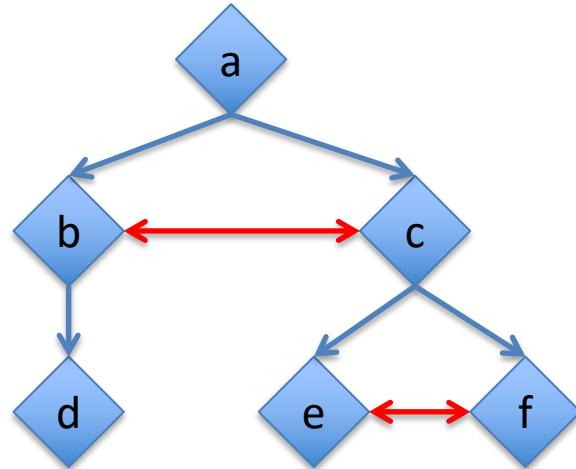


DifferentFrom

```
hasChild(?x, ?y), hasChild(?x, ?z), DifferentFrom (?y, ?z)  
-> hasSibling(?y, ?z)
```

⚠️ works only if

```
DifferentIndividual(b, c)  
DifferentIndividual(e, f)
```



Example

```
hasChild(?x, ?y) -> hasDescendant(?x, ?y)  
hasChild(?x, ?y), hasDescendant(?y, ?z) -> hasDescendant(?x, ?z)
```

The screenshot shows the Protégé ontology editor interface with the individual 'a' selected. The left sidebar displays the class hierarchy, showing 'owl:Thing' as the root and 'Person' as a subclass. The main panel is divided into several tabs:

- Annotations:** Shows annotations for individual 'a'.
 - Annotations: a
- Rules:** Shows inference rules:
 - hasChild(?x, ?y), hasChild(?y, ?z) -> hasGrandChild(?x, ?z)
 - hasChild(?x, ?y) -> hasDescendant(?x, ?y)
 - hasChild(?x, ?y), hasDescendant(?y, ?z) -> hasDescendant(?x, ?z)
 - hasChild(?x, ?y), hasChild(?x, ?z), DifferentFrom (?y, ?z) -> hasSibling(?y, ?z)
- Description:** Shows types and descriptions for individual 'a'.
 - Types: Person
 - Same Individual As: b, c, d, e, f
 - Different Individuals: b, c, d, e, f
- Property assertions:** Shows object property assertions for individual 'a'.
 - Object property assertions:
 - hasChild c
 - hasChild b
 - hasDescendant d
 - hasDescendant e
 - hasDescendant f
 - hasDescendant b
 - hasDescendant c
 - hasGrandChild d
 - hasGrandChild e
 - hasGrandChild f

DL-safe rules

Query answering for DL-axioms + rules is **undecidable**

It is **decidable** if rules are DL-safe

A rule r is called DL-safe if each variable in r occurs in a non-DL-atom in the rule body.

Practically: the variables in rules can only be bound to known individuals

Axioms:

TBox: $\text{Parent} \equiv \text{hasChild} \text{ some Person}$

ABox: $\text{Parent(a), Parent(b), Parent(c), Person(d), hasChild(a,d)}$

Rule:

$\text{hasChild(?x, ?y)} \rightarrow \text{PersonWithChild(?x)}$

consequence:

$\text{PersonWithChild(a)}$

without the DL-safe restriction:

$\text{PersonWithChild(a), PersonWithChild(b), PersonWithChild(c)}$

Builtin predicates

To deal with numbers, strings, etc.

```
Rectangle(?x), hasWidthInMetres(?x, ?w), greaterThan(?w, 10)
-> WideRectangle(?x)
```

```
Rectangle(?x), hasHeightInMetres(?x, ?h), hasWidthInMetres(?x,
?w), greaterThan(?a, 100), multiply(?a, ?w, ?h)
-> LargeRectangle(?x)
```

swrlb:equal
swrlb:notEqual
swrlb:lessThan
swrlb:lessThanOrEqual
swrlb:greaterThan
swrlb:greaterThanOrEqual

swrlb:add
swrlb:subtract
swrlb:multiply
swrlb:divide
swrlb:integerDivide
swrlb:mod
swrlb:pow
swrlb:unaryPlus
swrlb:unaryMinus
swrlb:abs
swrlb:ceiling
swrlb:floor
swrlb:round
swrlb:roundHalfToEven
swrlb:sin
swrlb:cos
swrlb:tan

swrlb:stringEqualIgnoreCase
swrlb:stringConcat
swrlb:substring
swrlb:stringLength
swrlb:normalizeSpace
swrlb:upperCase
swrlb:lowerCase
swrlb:translate
swrlb:contains
swrlb:containsIgnoreCase
swrlb:startsWith
swrlb:endsWith
swrlb:substringBefore
swrlb:substringAfter
swrlb:matches
swrlb:replace
swrlb:tokenize

When you don't need SWRL: DL rules

Some SWRL rules can be encoded in OWL expressions

Example

Man(?x) \wedge hasBrother(?x,?y) \wedge hasChild(?y,?z) \rightarrow Uncle(?x)

becomes

Man \sqcap \exists hasBrother. \exists hasChild.T \sqsubseteq Uncle

it's sometimes tricky ...

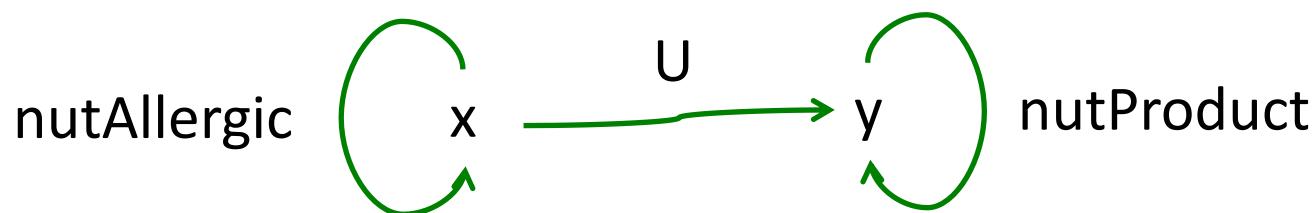
$\text{NutAllergic}(x) \wedge \text{NutProduct}(y) \rightarrow \text{dislikes}(x,y)$

$\text{NutAllergic} \equiv \exists \text{nutAllergic}. \text{Self}$

$\text{NutProduct} \equiv \exists \text{nutProduct}. \text{Self}$

$\text{nutAllergic} \circ \mathbf{U} \circ \text{nutProduct} \sqsubseteq \text{dislikes}$

\mathbf{U} = universal property ($x \mathbf{U} y$ is always true)

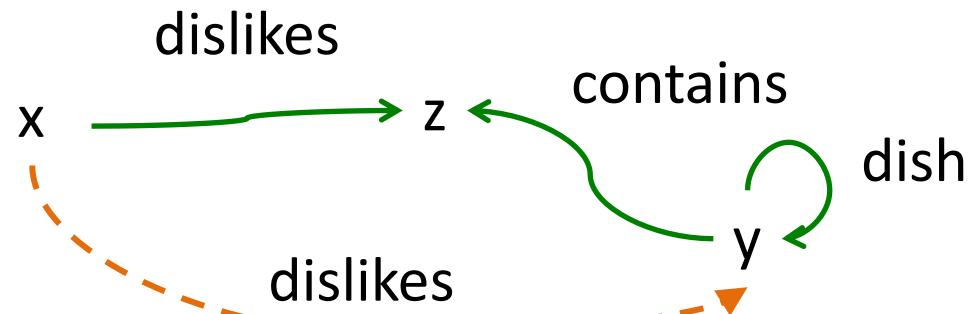


... more

$$\text{dislikes}(x,z) \wedge \text{Dish}(y) \wedge \text{contains}(y,z) \rightarrow \text{dislikes}(x,y)$$

becomes

- $\text{Dish} \equiv \exists \text{dish}. \text{Self}$
- $\text{dislikes} \circ \text{contains}^- \circ \text{dish} \sqsubseteq \text{dislikes}$



Rules vs. SPARQL queries

- Rules are “executed” globally
 - all rules must be satisfied simultaneously
- Rules may have interactions
 - the outcome of a rule may trigger another one
- SPARQL queries are executed independently

Simulating rules with queries

define a 'construct' query for each rule

repeat

- execute each query
- add the results to the RDF graph

until nothing new is created

$\text{parent}(\text{?x}, \text{?y}) \wedge \text{ancestor}(\text{?y}, \text{?z}) \rightarrow \text{ancestor}(\text{?x}, \text{?z})$

construct {?x ancestor ?z.}

where {?x parent ?y. ?y ancestor ?z.}

Simulating rules with queries

$\text{parent}(\text{x}, \text{y}) \wedge \text{ancestor}(\text{y}, \text{z}) \rightarrow \text{ancestor}(\text{x}, \text{z})$

repeat

```
  construct {?x ancestor ?z.}
  where {?x parent ?y. ?y ancestor ?z.}
```

until nothing new

Can be extremely inefficient