

An Introduction to Description Logics

1. Syntax and Semantics

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Contents

- Individuals, classes and properties
- Class constructors
- Ontologies
- Reasoning

The world according to DL

- There are **individuals**
- Individuals may be interconnected through **roles**
- Individuals belong to **concepts**

Similar to RDF/S (resources, properties, classes)

But different

- individuals, classes, and roles are disjoint
 - a class may not be an instance of another class
 - an individual may not have instances

DL languages

a family of logical languages with different

- class constructors
- role constructors

aims

- formalize the knowledge representation languages of the 80-90's (CLASSIC, KL-ONE, semantic networks, ...)
- decidable
 - there is an algorithm to check the consistency of any set of formulas
 - \Rightarrow not as expressive as First order logic

\mathcal{ALC} – concept constructors

Start with a vocabulary of **concept names** (N_C), **role names** (N_R) and **individual names** (N_O)

Concept expressions may be

- the top concept (everything): \top
- the bottom (impossible) concept: \perp
- a concept name: A
- a conjunction of concepts: $C \sqcap D$
- a disjunction: $C \sqcup D$
- a complement: $\neg C$
- an existential restriction: $\exists R.C$
- a universal restriction: $\forall R.C$

Manchester notation

class name

Building

conjunction

Student **and** Employee

disjunction

Man **or** Woman

complement

not Student

existential restriction

child **some** Student

universal restriction

child **only** Student

Semantics of \mathcal{ALC}

An interpretation I consists of

- a domain Δ^I
- an interpretation function I such that maps:
 - every *individual name* a to an element $I(a) \in \Delta^I$
 - every *concept* to a subset of Δ^I
 - every *role* to a binary relation on Δ^I (a subset of $\Delta^I \times \Delta^I$)

The semantics of non atomic concepts and roles is (recursively) defined in terms of atomic concept and role interpretations.

Semantics of concept expressions

$I(\perp)$	$= \emptyset$
$I(\top)$	$= \Delta^I$
$I(C \sqcap D)$	$= I(C) \cap I(D)$
$I(C \sqcup D)$	$= I(C) \cup I(D)$
$I(\neg C)$	$= \Delta^I - I(C)$
$I(\exists R.C)$	$= \{x \in \Delta^I : \exists y. (x, y) \in I(R) \text{ and } y \in I(C)\}$
$I(\forall R.C)$	$= \{x \in \Delta^I : \forall y. (x, y) \in I(R) \Rightarrow y \in I(C)\}$

Exercise

Consider the vocabulary

concepts: Human, Cat; roles: hasParent

and the interpretation

- $\Delta^I = \{a, b, c, d, e, f, k, l, m\}$
- $I(\text{Human}) = \{a, b, c, d, e\}$
- $I(\text{Cat}) = \{k, l, m\}$
- $I(\text{hasParent}) = \{(a, b), (a, c), (c, d), (b, e), (b, f), (k, m)\}$

What are the formal interpretations (and informal meaning) of

- $\text{Human} \sqcap \text{Cat}$
- $\neg \text{Human}$
- $\exists \text{ hasParent}.\top$
- $\exists \text{ hasParent}.\text{Cat}$

Exercise (cont.)

If

- $\Delta^I = \{a, b, c, d, e, f\}$
- $I(\text{Human}) = \{a, b, c, d, e\}$
- $I(\text{Cat}) = \{k, l, m\}$
- $I(\text{hasParent}) = \{(a, b), (a, c), (c, d), (b, e), (b, f), (k, m)\}$

what are the interpretations of

- $\forall.\text{hasParent.Cat}$
- $\exists \text{ hasParent}.\top \sqcap \forall.\text{hasParent.Cat}$
- $\exists \text{ hasParent} . (\exists \text{ hasParent.Human})$
- $\exists \text{ hasParent} . (\forall \text{ hasParent}.\bot)$

More class constructors (\mathcal{ALCOQ})

Number restrictions on properties

$$\geq n \text{ R.C}$$

$$\{x \in \Delta^I : \#\{y \mid (x, y) \in I(R) \text{ and } y \in I(C)\} \geq n\}$$

$$\leq n \text{ R.C}$$

$$\{x \in \Delta^I : \#\{y \mid (x, y) \in I(R) \text{ and } y \in I(C)\} \leq n\}$$

$$= n \text{ R.C} \leftrightarrow \geq n \text{ R.C and } \leq n \text{ R.C}$$

Enumeration of individuals

$$\{i_1, i_2, \dots, i_n\}$$

$$\{x \in \Delta : x = I(i_1) \text{ or } x = I(i_2) \text{ or } \dots, \text{ or } x = I(i_n)\}$$

DL Knowledge Base

Vocabulary:

class names, property names, individual names

Terminological axioms (TBox):

provide class definitions and relationships between classes

Role axioms (RBox):

about roles

Assertional axioms (ABox):

about individuals

Terminological Axioms (TBox)

Axioms of the form

$$\begin{aligned} C &\sqsubseteq D \\ C &\equiv D \\ C &\text{disjoint } D \end{aligned}$$

Axiom satisfaction by an interpretation I (notation: $I \models \text{Axiom}$)

$$\begin{aligned} I \models C &\sqsubseteq D \text{ if and only if } I(C) \subseteq I(D) \\ I \models C &\equiv D \text{ if and only if } I(C) = I(D) \\ I \models C &\text{disjoint } D \text{ if and only if } I(C) \cap I(D) \neq \emptyset \end{aligned}$$

Exercises

Find an interpretation I of the vocabulary

- `Cat`, `Mammal`, `Human` (concept names),
- `hasParent` (role names),
- `felix`, `bob`, `alice` (individual names)

that has $\Delta^I = \{a, b, c, d, e, f\}$ and satisfies the axioms

- $\text{Cat} \sqsubseteq \text{Mammal}$
- $\text{Human} \sqsubseteq \text{Mammal}$
- $\text{Human} \sqsubseteq \forall \text{hasParent}.\text{Human}$
- $\{\text{bob}, \text{alice}\} \sqsubseteq \text{Human}$
- $\{\text{felix}\} \sqsubseteq \text{Cat}$
- $\text{Cat} \sqsubseteq \exists \text{hasParent}.\top$
- $\top \sqsubseteq \text{Mammal}$

Axioms on roles (RBox) (*ALCHOQ*)

$$P \sqsubseteq R$$

if a is linked to b through P
then a is linked to b through R

$$I \models P \sqsubseteq R \text{ if and only if } I(P) \subseteq I(R)$$

Examples

- $\text{mother} \sqsubseteq \text{parent}$
- $\text{primaryFunction} \sqsubseteq \text{function}$

Axioms on roles: property chains

$$R_1 \circ R_2 \circ \dots \circ R_n \sqsubseteq P$$

if **a** is linked to **b** through a property chain $R_1 \circ R_2 \circ \dots \circ R_n$
then **a** is linked to **b** through P

$$I \models R_1 \circ R_2 \circ \dots \circ R_n \sqsubseteq P$$

if and only if

$$\forall y_0, y_1, \dots, y_n : (y_0, y_1) \in I(R_1), \dots, (y_{n-1}, y_n) \in I(R_n) \rightarrow (y_0, y_n) \in I(P)$$

Examples

- parent \circ parent \sqsubseteq grandParent
- parent \circ parent \circ child \circ child \sqsubseteq cousinOrSiblingOrSelf

More axioms on roles

functional(R)

$$(x, y) \in I(R) \text{ and } (x, z) \in I(R) \rightarrow y = z$$

inverse(R, S)

$$(x, y) \in I(R) \rightarrow (y, x) \in I(S)$$

symmetric(R)

$$(x, y) \in I(R) \rightarrow (y, x) \in I(R)$$

transitive(R)

$$(x, y) \in I(R) \text{ and } (y, z) \in I(R) \rightarrow (x, z) \in I(R)$$

reflexive(R)

$$\forall x \in \Delta^I : (x, x) \in I(R)$$

Examples

`functional(biologicalMother)`

`inverse(mother, child)`

`symmetric(friend)`

`transitive(before)`

`reflexive(knows)`

Assertions on individuals (ABox)

Axioms asserting that

- a named individual belongs to a class

$C(i)$

$I \models C(i)$ if and only if $I(i) \in I(C)$

- a named individual is linked to another one through a role

$R(i, j)$

$I \models R(i, j)$ if and only if $(I(i), I(j)) \in I(R)$

Assertions on individuals (ABox)

Axioms asserting that

- two named individuals are different

$i \text{ differentFrom } j$

$I \models i \text{ differentFrom } j$ if and only if $I(i) \neq I(j)$

- two named individuals are equal

$i \text{ sameAs } j$

$I \models i \text{ sameAs } j$ if and only if $I(i) = I(j)$

Exercise

Find a minimal interpretation that satisfies

- `Man(bob)`
- `Woman(lisa)`
- `Human(sam)`
- `Man \sqcup Woman \sqsubseteq Human`
- `Man disjointFrom Woman`
- `hasSibling(bob, lisa)`
- `symmetric(hasSibling)`
- `father(lisa, max)`
- `father(lisa, mix)`
- `mix differentFrom sam`
- `functional(father)`