

## EEG CORTICAL IMAGING: A VECTOR FIELD APPROACH FOR LAPLACIAN DENOISING AND MISSING DATA ESTIMATION

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### ABSTRACT

The surface Laplacian is known to be a theoretical reliable approximation of the cortical activity. Unfortunately, because of its high pass character and the relative low density of the EEG caps, the estimation of the Laplacian itself tends to be very sensitive to noise.

We introduce a method based on vector field regularization through diffusion for denoising the Laplacian data and thus obtain robust estimation. We use a forward-backward diffusion aiming for source energy minimization while preserving contrasts between active and non-active regions. This technique uses headcap geometry specific differential operators to counter the low sensor density. The comparison with classical denoising schemes clearly demonstrates the advantages of our method.

We also propose an algorithm based on the Gauss-Ostrogradsky theorem for estimation of the Laplacian on missing (bad) electrodes, which can be combined with the regularization technique in order to provide a joint validation framework.

### 1. INTRODUCTION

In the context of Brain Machine Interface projects, researchers are interested in properly distinguishing between different mental activities. A promising way of approaching this problem is to identify the active brain areas, either through the EEG inverse problem solution, or through cortical imaging.

In the EEG inverse problem one attempts to recover the electrical sources which are at the origin of the measured EEG data, either by performing dipole localization (usually nonlinear search for the best fit), or distributed linear inversion, that is discretizing the whole solution space (gray matter volume) and estimating the source amplitudes in each solution point.

Cortical imaging does not attempt to resolve this highly ill-posed problem, but merely to identify the active areas of the cortex through estimation of the potential distribution on the cortical surface. It can be proven that the surface Laplacian of the EEG data can be used as a reliable approximation of the cortex potential distribution.

Known techniques for Laplacian estimation involve the use of analytical derivation via interpolating approximations (e.g. spline) of the potential data [2], [3]. However these approaches do not only fit to the valuable potential data, but also to noise. Combined with the known high-pass character of the Laplacian

filter, even with low variance noise these methods can generate unstable results. This paper focuses on robust estimation of the Laplacian EEG from data collected on a typical electrode cap.

### 2. PHYSICAL CONSIDERATIONS

The measured EEG potential is a sum of contributions from all the active sources in the brain. The deep sources tend to contribute more uniformly to all electrodes, as the distances to each electrode are on the same magnitude order. On the other hand, cortical sources tend to influence only the closest electrodes. In terms of spatial frequencies, the deep sources will produce the low band frequencies, while the cortical sources count for the high band frequencies.

If we want to study only the cortical sources we need to apply some kind of high-pass spatial filtering. Following the works from [1], [2] it was proven that the Laplacian filter is a legitimate filter for cortical imaging.

Indeed, from the Laplace equation:

$$\Delta \phi = -\nabla \cdot \vec{E} = -\nabla \cdot (\sigma_{scalp} \vec{j}) = 0 \quad (1)$$

$\phi$ ,  $\vec{E}$ ,  $\vec{j}$ ,  $\sigma_{scalp}$  stand for the electric field potential, field intensity, current intensity and scalp conductivity respectively. Equation 1 is derived using the standard relationships:

$$\vec{E} = -\nabla \phi ; \vec{E} = \sigma \vec{j} ; \Delta = \underbrace{\nabla \cdot (\nabla)}_{\text{Laplacian}} = \underbrace{\text{div}}_{\text{grad}}$$

Splitting the current into the surface orthogonal and parallel components, one obtains:

$$\Delta_s \phi = \sigma_{scalp} \nabla \cdot \vec{j}_{\parallel} = -\sigma_{scalp} \nabla \cdot \vec{j}_{\perp} \quad (2)$$

The surface Laplacian is thus proportional to the divergence (derivative with respect to the orthogonal axis) of the surface perpendicular electric current. Neglecting the outgoing electric current (the air conductivity is practically null), and the scalp potential with respect to the cortical potential (due to huge attenuation in the skull), we derive :

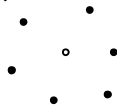
$$\begin{aligned} \nabla \cdot \vec{j}_{\perp} &\cong \frac{\vec{j}_{\perp,skull} - \vec{0}}{d_{scalp}} \cong \frac{1}{d_{scalp}} \frac{\phi_{scalp} - \phi_{cortex}}{\sigma_{skull} d_{skull}} \\ \Delta_s \phi &\cong \frac{1}{d_{scalp} d_{skull}} \frac{\sigma_{scalp}}{\sigma_{skull}} \phi_{cortex} \end{aligned} \quad (3)$$

The Laplacian filter therefore produces an image *proportional to the cortical potentials*. To our knowledge the previous relationship has been first presented in [1].

### 3. GEOMETRY AND DIFFERENTIAL OPERATORS

Unfortunately the Laplacian computation involves the use of differential operators. Since in general the EEG standard caps are not regularly spaced, this has pushed researchers to avoid differential operators for direct Laplacian estimation, in favor of analytical derivation from spline interpolators. We present here a method for determining specific differential operators for any irregular cap, provided that it is dense enough (roughly more than 100 electrodes).

We use the concept of *neighborhood* (Figure 1), i.e. for each electrode we search for its nearest neighbors (either manually or automatically).



**Figure 1. Typical electrode neighborhood**

We are looking for the derivatives of a function on a surface. Assuming that the surface is locally spherical, we will look for the derivatives with respect to the orthogonal unitary vectors which form the tangent plane to this surface at the position of the considered electrode. We need to evaluate the partial derivatives of a function  $f$  such as to respect as much as possible the Equation 4, for every neighboring electrode  $el_i$ .

$$f(el_i) - f(el_0) = \begin{pmatrix} \partial f / \partial x_1 & \partial f / \partial x_2 \end{pmatrix} \begin{pmatrix} x_1(el_i) \\ x_2(el_i) \end{pmatrix} \quad (4)$$

$el_0$  denotes the central electrode and the orthogonal system formed by the unitary vectors  $(\mathbf{x}_1, \mathbf{x}_2)$  is supposedly originated, without any loss of generality, in the central electrode. This is clearly an overdetermined problem that can be solved using the classical least squares solution (Equation 5).

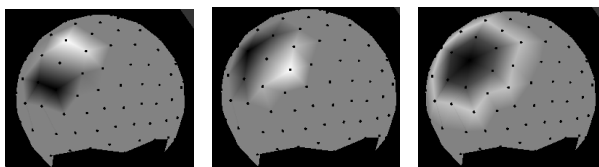
$$\frac{\partial f}{\partial \mathbf{x}} = (X'X)^{-1} X'f \quad (5)$$

Where  $\left(\frac{\partial f}{\partial x_i}\right) = \frac{\partial f}{\partial x_i}$ ,  $X_{ij} = x_j(el_i)$ ,  $f_i = f(el_i) - f(el_0)$

and  $t$  stands for squared.

The kernel of this partial derivative operator is independent of the values of the function, and it is exactly the gradient operator. The divergence operator can be obtained by applying this kernel twice, once for each component of the vector field (in the same local coordinate system) and retaining only the necessary terms (diagonal). The determination of the Laplacian operator is then straightforward.

Since the operators are local, they need to be rotated for coherence on the surface, for instance they can be aligned with the local system of the vertex electrode. Figure 2 shows a typical image of the gradient (left and middle image) and the Laplacian (right image) operators on a spherical head shape with a 123 electrode cap.



**Figure 2. Gradient and Laplacian operators**

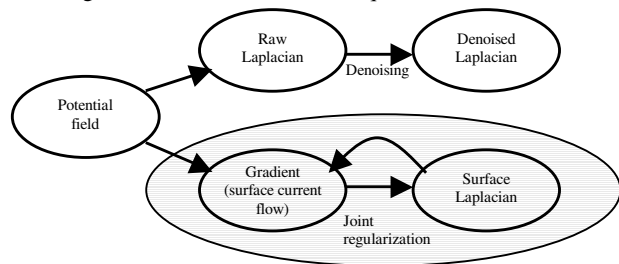
Positive values are mapped continuously to white-gray and negative values to black-gray. The operators show standard characteristics: orthogonality of the gradient operators and isotropy of the Laplacian. The operators are aligned with the coordinate system of the vertex electrode.

The same kind of patterns are reproduced for all electrodes, except for the border ones, where it is not possible to obtain a reliable Laplacian estimate (second order derivative on a border), but where it is nevertheless possible to obtain a correct approximation of the gradients.

### 4. LAPLACIAN DRIVEN DIFFUSION REGULARIZATION

Applying the Laplacian operator previously defined is the first step in our search for a robust Laplacian estimator. The least squares solution is the Maximum Likelihood estimator for Gaussian noise, so from its very conception our operator has, up to some degree, embedded noise immunity. Experiences however show that the noise sensitivity of the Laplacian surpasses its immunity. Denoising the Laplacian is thus required.

The natural way to do it would be directly denoising the raw Laplacian (upper path in Figure 3). We believe this to be unsatisfactory, therefore we will work instead on the surface gradient field using Laplacian priors, which implies joint regularization of the gradient and of the Laplacian (lower path of in Figure 3). Furthermore, this allows us to embed physical knowledge into the basis of our technique.



**Figure 3. Denoising schemes**

The surface gradient field is nothing else than the electric field intensity, or equivalently the current flow on the surface. Rigorously, the sources of this flow are expressed by the surface Laplacian, since its divergence is proportional to it (Equation 2). We start then the construction of our algorithm by stating the **source energy minimization** principle in Equation 6.

$$\text{minimize } \|\Delta_s\|^2 \quad (6)$$

In the same time we need to insure closeness to the original data  $\vec{j}_0$ . Using the Lagrangian multipliers method written in an energy form we obtain ( $\Omega$  is the Lagrangian multiplier):

$$\text{minimize } \frac{1}{2} \left( \Omega \|\vec{j} - \vec{j}_0\|^2 + \|\Delta_s\|^2 \right) \quad (7)$$

Since this type of approach tends to oversmooth and/or not sufficiently eliminate noise (the first term is bound to keep noise components and the second one to oversmooth), the solution is to *smooth out noise* and *a posteriori restore and accentuate* the boundaries of the vector field image. The process will thus be overall anisotropic, rendering an uniform field in the no or low source (Laplacian) regions, and suppressing noise without reducing source contrasts in active regions.

This type of anisotropic behavior perfectly fits diffusion regularization schemes, which have already been proven to yield excellent results with vector images [4], [5]. They are iterative gradient-descent-like methods based on the Euler-Lagrange minimum-effort (least-action) principle, which states that any physical system will follow a path such as to minimize the Lagrangian functional over time:

$$\text{minimize} \int_t L(x(t), \dot{x}(t)) dt$$

Historically, in mechanics, the Lagrangian functional  $L$  stands for the energy of the system,  $x$  for its position,  $t$  for time, and the point above  $x$  for its time derivative, that is the speed. The equilibrium state of this system is indicated by:

$$\frac{\partial L}{\partial x} - \frac{\partial}{\partial t} \frac{\partial L}{\partial \dot{x}} = 0 \quad (8)$$

This principle is usually applied in image processing by switching the time variable with the image coordinates and the Lagrangian with a functional of the form:

$$\frac{\alpha}{2} (I - I_0)^2 + \Psi(\|\nabla I\|) \quad (9)$$

where  $I$  stands for the scalar image intensity values,  $\alpha$  is a Lagrangian multiplier and  $\Psi$  a preferably convex continuous function. The equilibrium (and the descent direction) is given by:

$$\partial I = \alpha(I_0 - I) + \nabla \cdot \left( \frac{\Psi'(\|\nabla I\|)}{\|\nabla I\|} \nabla I \right) \quad (10)$$

For vector valued images, like the EEG gradient field, the Equation 10 cannot be applied directly. It would be possible to perform diffusion component-wise, but this would lead to uncoupling. Instead, we can couple the diffusion using an unified measure for the gradient norm, based on the tensor gradient. Using the notations of Equation 6 we will thus have :

$$\partial \bar{J} = \Omega(\bar{J}_0 - \bar{J}) + \nabla \cdot \left( \underbrace{\frac{\Psi'(\|\nabla \bar{J}\|)}{\|\nabla \bar{J}\|}}_{F_\Psi} \nabla \bar{J} \right) \quad (11)$$

The divergence is to be applied separately for each component. We still need to define a norm for the tensor gradient and to find the proper function  $\Psi$ .

For natural images, and in order to preserve borders, this function is chosen such as that the behavior function  $F_\Psi$  (Equation 11), which dictates how the diffusion acts in the presence of gradients, is monotonically decreasing. The gradient norm is usually selected as a combination of the eigenvalues of the gradient tensor. We already have a proper and convenient norm definition, that is the absolute value of the gradient tensor trace. Indeed, in every cartesian system  $(\mathbf{x}_1, \mathbf{x}_2)$ :

$$(\nabla \bar{J})_{ij} = \frac{\partial J_i}{\partial x_j} \stackrel{\text{def}}{\Rightarrow} \|\nabla \bar{J}\| = \left| \text{Tr}(\nabla \bar{J}) \right| = \frac{1}{\sigma_{scalp}} \|\Delta_s\| \quad (12)$$

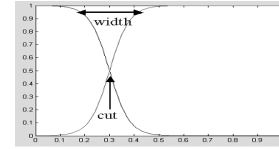
The conductivity can be regrouped with the Lagrangian multiplier and we naturally choose the  $\Psi$  function as the energy functional:

$$\Psi(\|\Delta_s\|) = \frac{1}{2} \|\Delta_s\|^2 \Leftrightarrow F_\Psi = 1 \quad (13)$$

We have converted the minimization problem defined in Equation 7 into an uniform diffusion process, which indifferently

affects low or high gradients. This is the first step of the iteration, the second one needing to restore and accentuate.

This can be done by using backward diffusion on high gradient regions. Backward diffusion acts in the same way as forward diffusion, but simply switching the evolution direction, which enables it not to smooth out gradients, but, on a contrary, accentuate them. In the same time normal forward diffusion can be also applied on low gradient regions. This forces the choice of a combined forward-backward diffusion process second step, with behavior functions that should look roughly as in Figure 4, where the horizontal axis is the normalized Laplacian norm.



**Figure 4. Forward-Backward behavior functions**

In practice we use sigmoid shapes in the transition area and fixed values of 0 and 1 outside. The width of the transition should be small enough to ensure differentiation between active and non-active regions and large enough to avoid discontinuity. The backward function is the increasing one.

The cut value  $\gamma$ , which fixes the border between active and non-active regions, is determined at each iteration using the Bienaymé-Tchebychev inequality, considering the noise  $n$  to be zero-mean, and that we want to insure a low probability  $P$  of noise appearance in the area to be restored:

$$P(|n| \geq \gamma) \leq \frac{\text{Var}(n)}{\gamma^2} \Rightarrow \gamma = \sqrt{\frac{\text{Var}(n)}{P}} \quad (14)$$

The noise variance is estimated as a global mean of local variances. The ratio between the restoration power (backward diffusion) and the forward diffusion,  $\beta$ , is given by the ratio between the power of the Laplacian before diffusion and the power of the present estimate, in order to avoid divergence (backward diffusing implies instability risks). In summary:

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Initialize  $\bar{J} = \nabla \phi$ 
Begin the iterations
  • uniform diffusion (Equations 10, 11, 13),  $\Omega = 0$ 
 $\bar{J} = \bar{J} + \tau \nabla \cdot (\nabla \bar{J})$ 
  • estimate  $\gamma$ , compute  $F_\Psi, B_\Psi, \beta$ .
 $\bar{J} = \bar{J} + \tau \nabla \cdot (F_\Psi \nabla \bar{J}) - \tau \beta \nabla \cdot (B_\Psi \nabla \bar{J})$ 
Repeat until convergence

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A few comments are necessary:

- $\tau$  is a step size, which should be linked with the geometry of the differential operators (see previous section).
- $\Omega = 0$ , the closeness to the original data is insured by initialization and restoration
- the stopping criterion is relative, which means that the iterations are stopped when the energy of the modifications is low when compared with the first iteration (typically 10%).
- the divergence operations are performed line by line.
- the Laplacian is easily computed as  $\Delta_s = \nabla \cdot \bar{J}$ .

## 5. PERFORMANCE TESTS

In order to evaluate the performance of the algorithm we have simulated a realistic setup. Inside a four-layer spherical head-model we generated dipole sources at random positions and with random orientations. The source grid corresponding with the gray matter and the electrodes positions of a 123 electrode cap were obtained from the HUG (Hopital Universitaire de Geneve, courtesy of R.Grave de Peralta and S. Gonzalez).

Using analytical expansions into Legendre polynomials we computed the ideal generated potentials and surface Laplacians directly from the source distribution. We added noise to the ideal potentials and performed direct estimation through the Laplacian operator, Laplacian driven diffusion regularization, classical filtering of the estimated Laplacian (median and Wiener filtering), and compared the results in terms of PSNR with the ideal surface Laplacians. Results shown in Figure 5 confirm the validity of our approach:

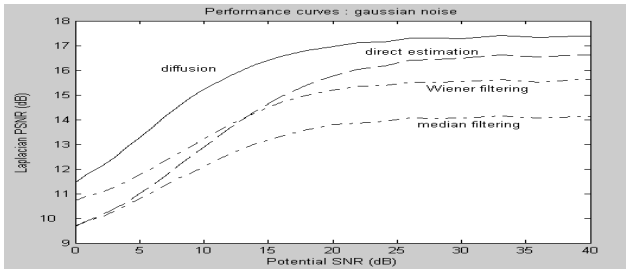


Figure 5. Performance curves

The randomization were carried out 10000 times for each SNR value (ranging from 0 to 40 dB and given with respect to the potential, where the noise was actually added), using three uniformly distributed sources (in space) with normal distributed amplitudes. Figure 5 shows the performances only for Gaussian noise, but the same type of curves ore observed for uniform and Laplacian noise. The 1 to 2 dB gain over any other method is mainly produced by the restoration procedure. Unfortunately the backward diffusion also had some unwanted influences, causing divergence (maximum number of iterations exceeded,  $NbMax = 10$ ) in roughly 1% of the simulations. The parameters involved in the iterations were set to:  $\tau = 0.01$ ,  $P = 0.1$ .

## 6. LAPLACIAN ESTIMATION ON MISSING (BAD) ELECTRODES AND JOINT VALIDATION

Suppose that one of the electrodes is malfunctioning (for instance the middle electrode of Figure 1). If reliable estimates of the neighborhood gradient field are available, the exact Gauss-Ostrogradsky theorem on a surface can be used to approximate Laplacian data on this electrode:

$$\iint_s (\nabla \cdot \vec{E}) ds = \int_c (\vec{E} \cdot \vec{n}) dl \quad (15)$$

The above formula actually states that the integral of the field divergence on a surface  $s$  closed by a contour  $c$  equals the value of the outgoing flow. Because of our discrete electrode system we can only approximate :

$$\Delta_s = \frac{\sum (\vec{E}_i \cdot \vec{n}) l_i}{S} \quad (16)$$

The computations for the contour vector  $\vec{n}_i$  and the contour length  $l_i$  for each electrode assume local constant curvature.

Equation 16 provides us with a joint validation framework: indeed, if the differential operators and the regularization procedures are physically sound, we should be able to recover the missing Laplacian data. As shown in Figure 6, the comparison with full data confirms our expectations.

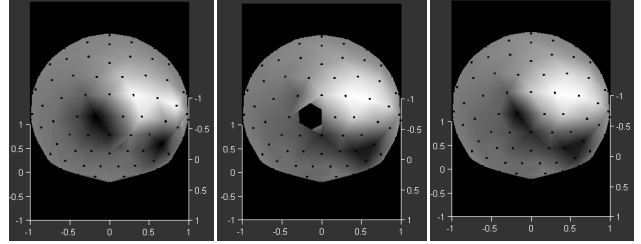


Figure 6. Joint validation

The left image in Figure 6 is the theoretical Laplacian. We have suppressed on purpose an electrode carrying important information from the potential data (hole in the middle image), regularized while completely ignoring it, and estimated the Laplacian on that electrode (right image).

## 7. CONCLUSIONS

We have presented a vector field approach for robust estimation of Laplacian data, leading to better discrimination between active/non-active regions, which should improve classification performance. We intend to pursue this approach and perform quantitative studies in a joint validation framework, using EEG data generated by Brain Machine Interface protocols.

Another important issue needs to be discussed, the inherent spatial smoothing effect caused by the low density of the EEG grid. Indeed, our estimated Laplacians are only averages over the surface defined by the neighbors. This effect accounts for the relatively low maximum PSNR value obtained. It is our goal to tackle this problem and continue towards source identification both on the cortical surface and in the inner brain.

## 8. REFERENCES

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