ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Synchronized Nets

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Introduction

- Internal versus External transitions

  -> pre-emptive power of internal transitions

- Synchronization expressions

  -> constraints
  -> abstractions

- Simplified framework: a single 'object'

Definitions

- (P,T,W) place/transition net
- A set of actions name
- μ_A labelling function: $\mu_A : T \rightarrow A \cup \{\tau\}$

  t is *internal* : $\mu_A(t) = \tau$, *external* otherwise

- $\mathbf{L_s}$ set of synchonizations

  $\mathbf{L_s}$ defined as:
  $\varepsilon \in \mathbf{L_s}$, (empty synchronization)
  $\forall\, a \in A: a \in \mathbf{L_s}$
  $\forall\, se, se' \in \mathbf{L_s}$: $se + se'$, $se \,.\, se'$, $se \,||\, se' \in \mathbf{L_s}$

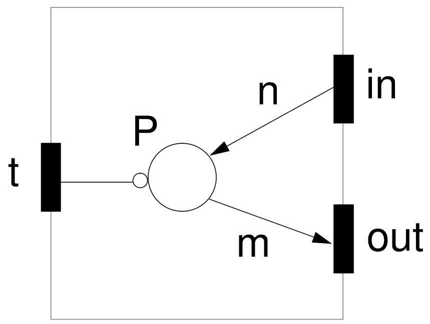
- Synchronized net (P, T, W, A, μ_a, μ_s), $\mu_s : T \rightarrow L_s$

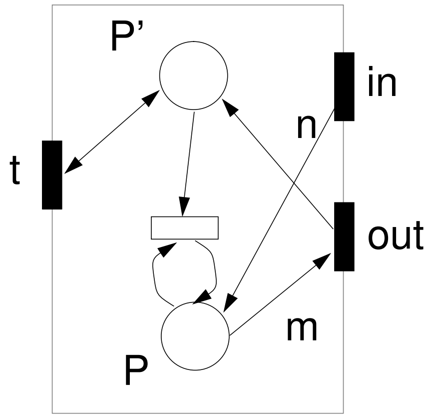
Classification

Petri Net:	μ_A injective $\forall\, t : \mu_s(t) = \varepsilon, \mu_A(t) \neq \tau$
Net with synchro:	$\forall\, t : \mu_A(t) \neq \tau$
Reactive Net	$\forall\, t : \mu_A(t) \neq \tau \Rightarrow \mu_s(t) = \varepsilon$
Reactive Net with synchro: no constraint	

	Internal: Synchro	Internal: No synchro	External: Synchro	External: No synchro
Petri Net				●
Net with Synchro			●	
Reactive Net		●	●	
Reactive Net with synchro	●		●	

Expressiveness





Petri Net with
Inhibitor Arcs
(-> Turing Machine)

<=>

Reactive Net

Net with Synchronizations
Inference rules

- Synchronization Part: $\Rightarrow_{sy} \subseteq \mathcal{M} \times L_s \times \mathcal{M}$

$\boxed{\varepsilon}$ $\quad \langle m, \varepsilon \rangle \Rightarrow_{sy} m$

$$\boxed{\mu} \quad \frac{\langle m - pre(t), \mu_s(t) \rangle \Rightarrow_{sy} m'}{\langle m, \mu_A(t) \rangle \Rightarrow_{sy} m' + post(t)} \quad pre(t) \subseteq m$$

$$\boxed{\|} \quad \frac{\langle m_1, s \rangle \Rightarrow_{sy} m' \quad \langle m_2, s \rangle \Rightarrow_{sy} m''}{\langle m_1 + m_2, s \| s' \rangle \Rightarrow_{sy} m' + m''}$$

$$\boxed{+^1} \quad \frac{\langle m, s \rangle \Rightarrow_{sy} m'}{\langle m, s + s' \rangle \Rightarrow_{sy} m'} \qquad \boxed{+^2} \quad \frac{\langle m, s' \rangle \Rightarrow_{sy} m'}{\langle m, s + s' \rangle \Rightarrow_{sy} m'}$$

$$\boxed{\cdot} \quad \frac{\langle m, s \rangle \Rightarrow_{sy} m_1 \quad \langle m_1, s' \rangle \Rightarrow_{sy} m'}{\langle m, s . s' \rangle \Rightarrow_{sy} m'}$$

Reactive Nets: inference rules

$=>_{sy} \subseteq \mathcal{M} \times L_S \times \mathcal{M}$ **and** $=>_{stab} \subseteq \mathcal{M} \times \mathcal{T} \times \mathcal{M}$

$\tau\text{-enab}(m) = \{t \in T \,/\, \mu_A(t) = \tau \text{ and } pre(t) \subseteq m\}$

- Synchronization Part

$\boxed{\varepsilon}$ $\quad <m, \varepsilon> =>_{sy} m$

$$\boxed{\mu} \quad \frac{<m\text{-}pre(t), \mu_s(t)> =>_{sy} m_1\ ,\ <m_1+post(t), \tau\text{-enab}(m_1+post(t))> =>_{stab} m'}{<m\ ,\ \mu_A(t)> =>_{sy} m'} \quad (pre(t) \subseteq m)$$

$$\boxed{\|} \quad \frac{<m_1, s> =>_{sy} m_1' \quad <m_2, s> =>_{sy} m_2' \quad <m_1'+m_2', \tau\text{-enab}(m_1'+m_2')> =>_{stab} m'}{<m_1+m_2\ ,\ s \,\|\, s'> =>_{sy} m'}$$

$\boxed{+ \text{ and } \bullet}$ similar

Reactive Nets (2)

$C(.,t) = post(t) - pre(t)$

- Stabilization Part

$\boxed{\varnothing}$ $\quad <m, \tau\text{-enab}(m)> =>_{stab} m \qquad (\tau\text{-enab}(m)=\varnothing)$

$$\boxed{\tau\text{-enab}} \quad \frac{<m+C(.,t), \tau\text{-enab}(m+C(.,t))> =>_{stab} m'}{<m\ ,\ \tau\text{-enab}(m)> =>_{stab} m'} \quad t \subseteq \tau\text{-enab}(m)$$

Reactive Net with Synchronizations

instable marking : internal transition t is enabled

reactive net: τ-enab(m)$\neq\emptyset$ => m instable

reactive net with synchronizations:

- local condition *(pre(t))*
- +
- synchronization part of t

Reactive Net with Synchronizations (2) inference rules

- First approximation

  -> introduction of a new symbol: $\perp$
  explicit notion of failure

- $\Rightarrow_{sy} \subseteq \mathcal{M} \times L_S \times \mathcal{M} \cup \{\perp\}$

  $\Rightarrow_{stab} \subseteq \mathcal{M} \times \mathcal{T} \times \mathcal{M}$

- Consideration of sub-Reactive-Net-with-Synchronizations may be necessary

Reactive Net with Synchronizations (3) inference rules

- Synchronization Part (1)

$\boxed{\varepsilon}$ $\quad <m, \varepsilon> \Rightarrow_{sy} m$

$\boxed{\mu_1}$ pre(t) $\subseteq$ m:

$$\frac{<m\text{-}pre(t),\ \mu_s(t)> \Rightarrow_{sy} m_1\ ,\ <m_1+post(t),\ \tau\text{-}enab(m_1+post(t))> \Rightarrow_{stab} m'}{<m\ ,\ \mu_A(t)> \Rightarrow_{sy} m'}$$

$\boxed{\mu_2}$ pre(t) $\subseteq$ m:

$$\frac{<m\text{-}pre(t),\ \mu_s(t)> \Rightarrow_{sy} \perp}{<m\ ,\ \mu_A(t)> \Rightarrow_{sy} \perp}$$

$\boxed{\mu_3}$ pre(t) $\not\subseteq$ m:

$<m\ ,\ \mu_A(t)> \Rightarrow_{sy} \perp$

Reactive Net with Synchronizations (4) inference rules

- Stabilization Part

$\boxed{\varnothing}$ $\quad <m, E> \Rightarrow_{stab} m \qquad (E=\varnothing)$

$\boxed{\tau_1}$ t $\in$ E

$$\frac{<m\text{-}pre(t),\ \mu_s(t)> \Rightarrow_{sy} m''\ ,\ <m''+post(t),\ \tau\text{-}enab(m''+post(t))> \Rightarrow_{stab} m'}{<m\ ,\ E> \Rightarrow_{stab} m'}$$

$\boxed{\tau_2}$ t $\in$ E

$$\frac{<m\text{-}pre(t),\ \mu_s(t)> \Rightarrow_{sy} \perp\ ,\ <m,\ E\backslash\{\tau\}> \Rightarrow_{stab} m'}{<m\ ,\ E> \Rightarrow_{stab} m'}$$

Reactive Net with Synchronizations (5)
inference rules

- Synchronization Part (2) : the ‘+’ operator

$$\boxed{+^{1}} \qquad \frac{<m, s> \Rightarrow_{sy} m'}{<m, s + s'> \Rightarrow_{sy} m'}$$

$$\boxed{+^{2}} \qquad \frac{<m, s> \Rightarrow_{sy} \perp \ , \ <m, s'> \Rightarrow_{sy} \perp}{<m, s + s'> \Rightarrow_{sy} \perp}$$

$$\boxed{+^{3}} \qquad \frac{<m, s> \Rightarrow_{sy} \perp \ , \ <m, s'> \Rightarrow_{sy} m'}{<m, s + s'> \Rightarrow_{sy} m'}$$

... and the symmetric companions w.r.t. s’

Reactive Net with Synchronizations (6)
inference rules

the operators ‘||’ and ‘.’ are difficult to handle

- || : reactive net (recall)

$$\boxed{\|} \qquad \frac{<m_1, s> \Rightarrow_{sy} m_1' \quad <m_2, s> \Rightarrow_{sy} m_2' \quad <m_1'+ m_2', \tau\text{-enab}(m_1'+ m_2')> \Rightarrow_{stab} m'}{<m_1+m_2 , s \| s'> \Rightarrow_{sy} m'}$$

- Reactive Net with Synchronisation
  -> problem of the explicit failure via ⊥
  -> dependency of the success related to the split of m into $m_1 + m_2$

Reactive Net with Synchronizations (5)
inference rules

- Synchronization Part (3) : the '||' operator

$\|^1$

$$\frac{\langle m_1, s\rangle \Rightarrow_{sy} \perp \ , \ \langle m_2, s'\rangle \Rightarrow_{sy} \perp}{\langle m_1+m_2, s \parallel s'\rangle \Rightarrow_{sy} \perp}$$

$\|^2$

$$\frac{\langle m_1, s\rangle \Rightarrow_{sy} m'_1 \ , \ \langle m_2, s'\rangle \Rightarrow_{sy} m'_2 \ , \langle m'_1+m'_2 \ , \tau\text{-enab}(m'_1+m'_2) \rangle \Rightarrow_{stab} m'}{\langle m_1+m_2, s \parallel s'\rangle \Rightarrow_{sy} m'}$$

... but nothing is said when either 'm_1 or m_2' succeeds and the other fails ('m_2 or m_1') because the failure may be due to 'a bad' splitting of m. So it musn't be concluded that the whole synchronization s||s' fails.

The following cases are the only exceptions

Reactive Net with Synchronizations (6)
inference rules

Let $m_e \in \mathcal{M}$ defined by $\forall$ p $\in$ P $m_e(p)=0$

- Synchronization Part (4) : the '||' operator

$\|^3$

$$\frac{\langle m_e, s\rangle \Rightarrow_{sy} m' \ , \ \langle m, s'\rangle \Rightarrow_{sy} \perp}{\langle m, s \parallel s'\rangle \Rightarrow_{sy} \perp}$$

$\|^4$

$$\frac{\langle m, s\rangle \Rightarrow_{sy} \perp \ , \ \langle m_e, s'\rangle \Rightarrow_{sy} m'}{\langle m, s \parallel s'\rangle \Rightarrow_{sy} \perp}$$

these rules handle the cases when either the synchronization s or s' is trivially satisfied.

Reactive Net with Synchronizations (7) inference rules

- The synchronization part involving the operator '.' has not yet been resolved.

-> The difficulty stems from the fact in the synchronization s.s', the failure of the first part, i.e. s, does not necessarily induces the failure of the whole. Indeed, this may come 'a bad' choice w.r.t. a previous non-deterministic choice. See for instance the rule page 12 (τ_2). All the possible solutions are to check before to conclude of the failure. In other word the following rule is false:

$$\cancel{\frac{\langle m, s\rangle \Rightarrow_{sy} m_1 \;,\; \langle m_1, s'\rangle \Rightarrow_{sy} \perp}{\langle m, s\, .\, s'\rangle \Rightarrow_{sy} \perp}}$$

Conclusion

- The operator of synchronization '.' remained to be defined in reactive net with synchronizations
- Its definition may cause the reformulation of the other operators
- The validation of these rules are to be done in a first approximation with examples and finally formally