EUROPHYSICS LETTERS 5 march 1998

Europhys. Lett., **100** (6), pp. 1-∞ (1998)

A lattice Boltzmann model for particle transport and deposition

A. Masselot and B. Chopard

Computer Science Department, University of Geneva
CH-1221 Geneva 4, Switzerland

(received ; accepted)

PACS. 47.11+j – Computational methods in fluid dynamics.
PACS. 02.70Ns – Molecular dynamics and particle methods.
PACS. 05.40+j – Fluctuation phenomena, random processes, and Brownian motion.

Abstract. – Solid particles (such as snow or sand) can get eroded and transported by the wind, and little understanding has been achieved in this domain. We propose a description of the phenomenon in terms of a cellular automata and lattice Boltzmann model. Numerical simulations show that plausible mechanisms are sufficient to explain a wide range of deposition patterns occurring at different space scales. In particular, we reproduce the so-called ripples, that is oscillations of the deposition surface for which no formation mechanism is clearly established.

Introduction. –

The dynamics of solid particles erosion, transport and deposition due to the action of a steaming fluid plays a crucial role in sand dune formation, sedimentation problems and snow transport. This field remains rather empirical compared to other domains of science and experts do not all agree on the mechanisms involved in these process.

In what follows, we focus the discussion on the problem of snow erosion and deposition. Diverse patterns of accumulations occur [1, 2], with quite different characteristic sizes: they range from the small ripples (oscillations of a few centimeters over a flat surface) up to large wind slabs leeward a mountain crest (tens of meters). In addition to interesting academic problems, snow transport by wind raises important practical questions that strongly influences human activities in mountain or nordic areas: snowdrifts burying roads or buildings [3], wind slabs launching avalanches. Obstacles are built in such places so as to influence the wind pattern and modify the deposit. But, as no forecasting tool exists, their shape and location are rather empirical [1].

Phenomenologically, snow transport (*i.e.* erosion and deposition) has been divided in three main processes, each corresponding to a different scale [1]: (i) *Creeping:* particles are "rolling" on the snow surface or making very little jumps; (ii)*Saltation:* in the first half meter above the surface, snow particles have been observed to be ejected vertically and follow a ballistic trajectory [4]; (iii) *Suspension:* it accounts for transport over larger scales (often seen as white smoke over mountains crests).

To predict snow transport and obtain the snow erosion/deposition areas, both wind tunnel experiments [1] and numerical computations (using statistical methods or classical CFD tools [3, 5]) have been investigated.

The latter approach is technically quite difficult: on top of a turbulent Navier-Stokes equations model, snow is usually added in some *ad hoc* way. Creeping, saltation and suspension are included separately and most of the time, one or two of these processes are neglected, depending on the goal pursued.

Here we propose a new, radically different numerical approach, free of the above complications and based on the cellular automata and lattice Boltzmann methods [6, 7, 8]. Instead of solving a differential equation, we model the phenomena in terms of snow and wind "particles". A significant advantage of this approach is that boundary conditions are easily implemented, due to the level of description in terms of particles. Thus, a granular and "continuous" flow can be described within the same model. Full attention can be given to the basic phenomena and our result is a unified model in which the deposition patterns at different scales are all obtained with the same erosion-deposition mechanisms.

The wind model. –

In our approach, the wind is described with a Lattice Boltzmann (LB) model: populations of particles densities move synchronously, according to discrete time steps, along the links of a regular lattice. When bouncing into each other, these densities are redistributed among the lattice directions in such a way that mass and momentum are conserved.

LB fluids are typically described by the quantities $f_i(\mathbf{r}, t) \in [0, 1]$ indicating the probability of presence of a particle entering site $\mathbf{r}$ at time t, with a velocity $\mathbf{v}_i$ pointing along lattice direction i. In our case $i = 1, ..., 8$ designates the geographical directions E, NE, N, NW, W, SW, S and SE in a two-dimensional square lattice and $i = 0$ refers to a population of rest particles ($\mathbf{v}_0 = 0$).

LB fluids capture the main essence of fluid dynamics, in the sense that the local average density $\rho =< \sum_i f_i >$ and local average velocity field $\mathbf{u} = (1/\rho) < \sum_i f_i \mathbf{v}_i >$ obey, within certain limits, the Navier-Stokes equation. As opposed to cellular automata fluids [9], the LB approach guarantees Galilean invariance and a correct form of the pressure term.

The evolution equation for the f_i's we consider is the so-called BGK model [7]

$$f_i(\mathbf{r} + \tau \mathbf{v}_i, t + \tau) - f_i(\mathbf{r}, t) = \frac{1}{\xi} \left[f_i^{(0)}(\mathbf{r}, t) - f_i(\mathbf{r}, t) \right] \tag{1}$$

where τ is the time step, ξ the relaxation time and

$$f_i^{(0)} = a_i \rho + \frac{b_i}{v^2} \rho \mathbf{v}_i \cdot \mathbf{u} + \rho e_i \frac{u^2}{v^2} + \rho \frac{h_i}{v^4} v_{i\alpha} v_{i\beta} u_\alpha u_\beta \tag{2}$$

is the local equilibrium distribution which depends only on the actual local velocity field $\mathbf{u}$ and particle density ρ. The coefficients ξ, a_i, b_i, e_i and h_i are chosen so that the dynamics (1) yields a Navier-Stokes behavior [8, 6].

The parameter ξ is known to control the viscosity as $\nu = (1/6)(2\xi - 1)$. High Reynolds flows can be achieved if the viscosity is small enough, *i.e.* when $\xi \approx 1/2$. Unfortunately, numerical instabilities appears when ν decreases too much. To solve this problem, a subgrid technique has been proposed [10] in lattice BGK models, which consists of adjusting dynamically and locally the relaxation time ξ according to the local velocity gradients $\xi = \tau_0 + C \left[\partial_\beta u_\alpha + \partial_\alpha u_\beta\right]^2$, where C is the so-called Smagorinski constant and $\tau_0 = 1/2$ the bare relaxation time. The indices α and β label spatial coordinates and summation over repeated indices is assumed. Having an effective relaxation time (and thus an effective viscosity) allows one to extrapolate unresolved scales and simulate high Reynolds number flows ($\sim 10^6$)[10].

In this study we are interested in producing a turbulent flow in a semi-infinite space. The boundary at $z = 0$ is the ground level and the system has, in principle, no limit in the positive z direction (in practice, of course, we have to define a boundary which reproduces an open system). The wind blows parallel to the ground direction (the x-axis) and wind particles are accelerated rightwards on the left boundary. The average wind speed profile in such a turbulent flow is described by the relation [11] $u_x(z) = u_* \log(z/z_0)$ up to some height z_{max} where it reaches the unperturbed wind speed u_∞. The quantity z_0 is dependent of the ground roughness and u_* depends on the viscosity and u_∞.

The effect of the Smagorinski constant C is to adjust the resolution scale of the flow. A small C is appropriate to describe small developed eddies. Thus, in order to produce the correct above velocity profile, which requires a finer resolution near the ground, we propose to make C depend on the height. It decreases linearly from a value C_∞ to 0 as the ground level is approached. This technique should be viewed as a way to define the proper boundary condition (change of the relaxation time due to an obstacle), ensuring a log profile for the velocity field $\mathbf{u}$. The distance over which C varies has been determined to be four lattice spacings in the simulations presented here.

The snow model. –

The snow transport is obtained by adding solid particles on top of the lattice BGK wind model. We define $N_i(\mathbf{r}, t) \in \mathbb{N}$ as the number of particles entering site $\mathbf{r}$ at time t with velocity v_i. Snow particles can be injected in the simulation (snowfall) or eroded from the ground, deposited and transported according to the combined effect of gravity and local wind velocity.

In order to observe the generic features of erosion, transport and deposition, we restrict ourselves to what we identify as the most significant processes and express them in simple rules. The macroscopic behavior will be captured provided that these ingredients are indeed the relevant ones and that enough scale separation exist between the different levels of description. We will show that simple processes are sufficient to obtain realistic deposition patterns through different space scales. The rules we consider are:

Transport: an arbitrary number of snow grains may reside at each lattice site. During the updating step, they synchronously move to the nearest neighbor sites. Between times t and $t + \tau_s$, particles at site $\mathbf{r}$ should move to $\mathbf{r} + \tau_s \mathbf{w}$, where τ_s is the time step associated to solid particle motion, $\mathbf{w} = \mathbf{u} + \mathbf{u}_{fall}$, with $\mathbf{u}$ the local wind speed and $\mathbf{u}_{fall}$ the falling speed (accounting for gravity). Usually, $\mathbf{r} + \tau_s \mathbf{w}$ does not correspond to a lattice node (see fig. 1), and the amount of grains that reach each neighbor is computed according to the following randomized algorithm, which ensures that the average motion is correct. One computes $p_i = \max(0, (\tau_s/\tau)(\mathbf{v}_i \cdot \mathbf{w})/|\mathbf{v}_i|^2)$, for $i = 1, 3, 5, 7$ (if $p_i > 0$, then $p_{i+4} = 0$, since $\mathbf{v}_i = -\mathbf{v}_{i+4}$). For efficiency, we choose $\tau_s \geq \tau$, but small enough so that p_i is always less than 1. Then, each particle ℓ jumps to site $\mathbf{r} + \mu_1^\ell \mathbf{v}_1 + \mu_3^\ell \mathbf{v}_3 + \mu_5^\ell \mathbf{v}_5 + \mu_7^\ell \mathbf{v}_7$, where μ_i^ℓ is a Boolean quantity which is 1 with probability p_i. If $N = \sum N_i$ is large enough, this binomial scattering can be approximated by a Gaussian distribution [12]. Note that in this algorithm, there is no attempt to include specific rules for creeping, saltation or suspension.

Deposition: lattice sites can be either solid (original landscape or deposited snow) or free (air). Snow particles on a free site may "freeze" if the neighbor site i they want to jump to is a solid site: $N_{frozen} \to N_{frozen} + N_i$, $N_i \to 0$. When N_{frozen} exceed some pre-assigned threshold N_s, the site becomes solid and subsequent incoming wind particle will bounce back (hence defining a new ground profile). This threshold gives a way to assign some size to the

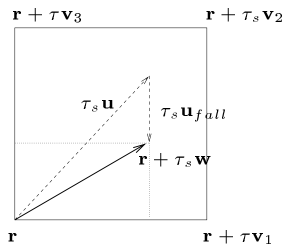

Fig. 1. – Transport, on the lattice, of particles in suspension in a fluid. A particle at site $\mathbf{r}$ jump to east with probability $p_1(1-p_3)$, to north-east with p_1p_3, to north with $p_3(1-p_1)$ and stay at rest with probability $(1-p_1)(1-p_3)$; (see text).

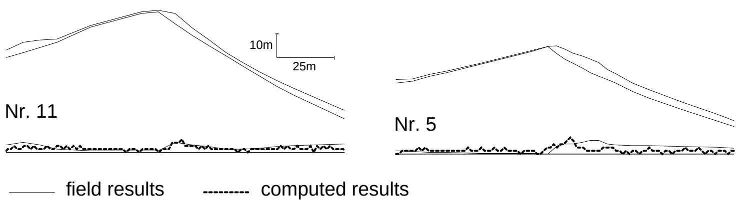

Fig. 2. – Snow deposit over mountains at different positions along the same crest – the Schwarzhorn-grat near Davos. The wind blows from left to right. Our results can be compared with terrain observation (crests labels refer to [14]). The lattice spacing is 1.5m, $N_s = 100$, $p = 0.08$ and C_∞ is made as small as possible to prevent numerical instabilities from developing: $C_\infty = 0.4$ on the left, and $C_\infty = 0.2$ on the right.

snow flakes. When a site solidifies the wind particles that may be present get trapped until erosion frees them again.

Erosion: deposited particles may be eroded under some conditions. For snow, the erosion rate seems to be related to the wind speed above the solid site [13], the concentration of snow being transported, the saturation concentration and the efficiency of the transport [1]. In our model, we express these mechanisms in a very simple way: erosion means that each frozen snow particle is ejected upwards ($N_3 \to N_3 + 1$, $N_{frozen} \to N_{frozen} - 1$) with probability p. When the local wind is fast enough, these ejected snow particles will be transported. Otherwise they fall back and freeze again.

Results. –

Since no first principle theory is available, the only and most convincing way to assess the validity of our model is to compare its predictions with the outdoor observations found in the literature. Our simulations has been tested against a wide range of situations and has shown to catch realistic phenomena at very different spatial scales by varying the Smagorinski constant C_∞, the threshold N_s and the erosion probability p. All the simulations are two-dimensional and have been performed on a massively parallel computer (a 8192 processors CM-200), which is very well suited to the present approach.

In fig. 2, the snow deposit over a mountain is shown for different positions along the crest. We observe the influence of the landscape profile on the deposit. A good agreement with field experiments is obtained [14].

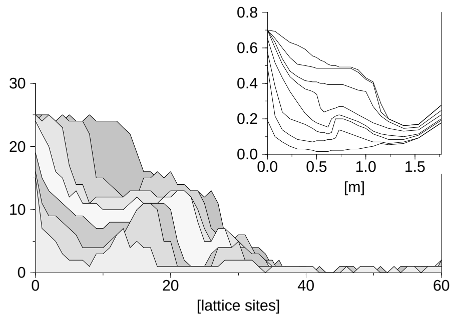

Fig. 3. – Deposition pattern of snow in a trench (0.7m$\times$1.7m, lattice spacing 0.03m, $N_s = 10$, $p = 0.04$ and $C_\infty = 0.3$). Snow particles are introduced on the left corner of the simulation; profiles are shown every 1000 iterations. The experimental profiles measured by Kobayashi [4] (given every 1/2 hour for the first layers) are sketched in the inset.

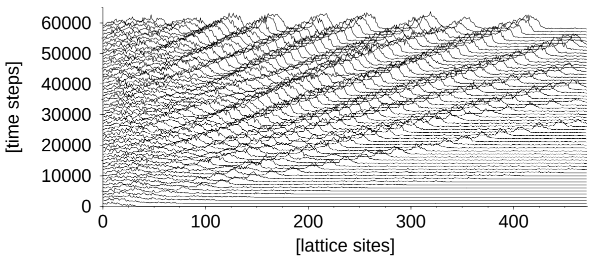

Fig. 4. – Formation of ripples, as obtained from our model. Particles are continuously injected in the lower left corner of the simulation and the ripples grow spontaneously. The deposition profile is given every 1000 time steps, which makes the horizontal ripple motion quite clear (as well as the higher speed of the smaller ripples "escaping" rightwards). The lattice spacing is around 0.03m, $N_s = 10$, $p = 0.02$ and $C_\infty = 5.0$.

Figure 3 illustrates the filling of a trench excavated in a large flat area. We may observe the non-intuitive deposition pattern and its time evolution where several growth regimes may be identified. Good qualitative agreement is observed between the model and reality [4], mainly for the first part of the experiment (growth of two deposition peaks) before the wind has slowed down in the outdoor experiment.

Finally, fig. 4 shows small scale patterns known as ripples occurring with both sand and snow transport. Ripples are mainly due to creeping transport. The ratio we find between the height and the spacing of the oscillations (called the *wave index*) ranges around 6; this value agrees with the lowest index found for sand [15] in field observations, fits well wind tunnel experiments values [16] and sand ripples in water [15]. Outdoor snow ripples are more complicated since freezing and cohesion have to be taken into account; their wave index has been measured to be around 16 [1, 17]. In agreement with real observations, we also see in our simulation that ripples move horizontally. This effect is illustrated in the figure. As observed in [18], our model also shows that large ripples can be built through the merging of smaller ones, traveling faster.

Our description of erosion and deposition uses three parameters: C_∞, N_s and p. Roughly speaking, these parameters give a way to select the characteristic scale of the patterns, but their precise role is still under investigation. If C_∞, N_s and p are badly chosen, the expected depositions pattern does not occur. For instance, ripples are obtained with larger p and C_∞ than the other two cases, because, as observed outdoor, they require a large snow mobility and weakly turbulent flow. The deposits over the crest has a large N_s because they correspond to a large scale problem.

In conclusion, our model not only produces quantitatively realistic deposits, it also provides, through simple and intuitive rules, a better understanding of the basic (and quite controversial) mechanisms that occur in particle transport. Various patterns of deposition result from the emergence of a collective effect rather than from mechanisms that have not yet been identified. Creeping, saltation or suspension are no longer three phenomena requiring each a special treatment: they are all captured by the same erosion/transport mechanisms. Our model results in an unified view of the basic laws governing the formation of particle deposition pattern.

***

We thank Dieter Issler and Peter Gauer from the Federal Institute for Snow and Avalanche Research in Davos for stimulating discussions. This research is supported by the Swiss National Science Foundation.

REFERENCES

[1] T. Castelle. *Transport de la neige par le vent en montagne: approche expérimentale du site du col du Lac Blanc.* PhD thesis, EPF Lausanne, Switzerland, 1995.

[2] U. Radok. Snow drift. *Journal of Glaciology*, 19(81):123–139, 1977.

[3] P.-A. Sundsbø and E.W.M. Hansen. Modelling and numerical simulation of snow drift around snow fences. In *ICSE-3,Sendai*, 1996.

[4] D. Kobayashi. Studies of snow transport in low-level drifting snow. *Contributions from the Institute of Low Temperature Science*, Series A(24):1–58, 1972.

[5] T. Uematsu, T. Nakata, K. Takeuchi, Y. Arisawa, and Y. Kaenada. Three-dimensional numerical simulation of snow drift. *Cold Regions Science Technology*, 20(1):25–39, 1991.

[6] B. Chopard and M. Droz. *Cellular automata modeling of physical systems.* Cambridge University Press, 1998. to appear.

[7] Y.H. Qian, D. d'Humieres, and P. Lallemand. Lattice BGK for Navier-Stokes equation. *Europhysics Letters*, 17(6):479–484, 1992.

[8] Y.H. Qian, S. Succi, and S.A. Orszag. Recent advances in lattice Boltzmann computing. In Dietrich Stauffer, editor, *Annual Reviews of Computationnal Physics III*, pages 195–242. World Scientific, 1996.

[9] U. Frish, B Hasslacher., and Y. Pommeau. Lattice-gas automata for the Navier-Stokes equation. *Phys. Rev. Lett.*, 56:1505, 1986.

[10] S. Hou, J. Sterling, S. Chen, and G.D. Doolen. A lattice subgrid model for high Reynolds number flows. *Fields Institute Communications*, 6:151–166, 1996.

[11] D.J. Tritton. *Physical fluid dynamics.* Clarendon Press, 1988.

[12] B. Chopard, L. Frachebourg, and M. Droz. Multiparticle lattice gas automata for reaction-diffusion systems. *Int. J. of Mod. Phys. C*, 5:47–63, 1994.

[13] M. Takeuchi. Vertical profile and horizontal increase of drift snow transport. *Journal of Glaciology*, 26(94):481–492, 1980.

[14] P.M.B. Föhn and R. Meister. Distribution of snow drifts on ridge slopes: measurments and theoretical approximations. *Annals of Glaciology*, 4:52–57, 1983.

[15] R.P. Sharp. Wind ripples. *Journal of Geology*, 71:617–636, 1963.

[16] H. Martinez. *Contribution à la modélisation du transport éolien de particules. Mesures de profiles de concentration en soufflerie diphasique.* PhD thesis, Grenoble I, february 1996.

[17] V. Cornish. *Waves of sand and snow.* T. Fisher Unwin, London, 1914.

[18] B.Y. Werner and D.T. Gillespie. Fundamentally discrete stochastic model for wind ripple dynamics. *Physical Review Letters*, 71:3230, 1993.