Epistemic semantics for rules in description logics

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Sources:

Franz Baader Werner Nutt. Basic Description Logics. Chapter in The Description Logics Handbook, Franz Baader et al. (Eds.)

Franz Baader Ralf Ku ¨sters Frank Wolter. Extensions to Description Logics. Chapter in The Description Logics Handbook, Franz Baader et al. (Eds.)

Operational semantics

Operationally, the semantics of a finite set R of trigger rules can be described by a forward reasoning process.

Starting with an initial knowledge base K, a series of knowledge bases K(0), K(1),… is constructed, where

- K(0) = K and
- K(i+1) is obtained from K(i) by adding a new assertion D(a) whenever
  - R contains a rule $C \Rightarrow D$
  - $K(i) \models C(a)$ holds
  - K(i) does not contain D(a)

- The result of the rule applications is a knowledge base K(n) that has the same TBox as K(0) and whose ABox is augmented by the membership assertions introduced by the rules.

- This procedural extension is independent of the order of rule applications.

- A set of trigger rules R uniquely specifies how to generate, for each knowledge base K, an extended knowledge base K^R.

- The semantics of a knowledge base K, augmented by a set of trigger rules, can thus be understood as the set of models of K^R.

The Meaning of Rules

An important difference between the trigger rule $C \Rightarrow D$ and the inclusion axiom $C \sqsubseteq D$ is that the trigger rule is not equivalent to its contrapositive $\neg D \Rightarrow \neg C$.

In addition, when applying trigger rules one does not make a case analysis.

> The inclusions $C \sqsubseteq D$ and $\neg C \sqsubseteq D$ imply that every object belongs to D,
>
> whereas none of the trigger rules $C \Rightarrow D$ and $\neg C \Rightarrow D$ applies to an individual a for which neither C(a) nor ¬C(a) can be proven.

Epistemic semantics

syntax rule to construct an epistemic concept:

$$\mathbf{K}C$$

KC denotes those objects for which the knowledge base knows that they are instances of C.

Trigger rules $C \Rightarrow D$ translation:

$$\mathbf{K}C \sqsubseteq D$$

Intuitively, the K operator has the effect that the axiom is only applicable to individuals that appear in the ABox and for which ABox and TBox imply that they are instances of C.

Epistemic principles

Principles assumed to govern the epistemic operator

- only true facts are known:

$$\mathbf{K}C \sqsubseteq C$$

  - if an object is known to be an instance of C, then it is an instance of C);
- positive introspection

$$\mathbf{K}C \sqsubseteq \mathbf{KK}C$$

  - if it is known that an object is an instance of C, then this is known;
- negative introspection

$$\neg\mathbf{K}C \sqsubseteq \mathbf{K}\neg\mathbf{K}C$$

  - if it is not known whether an object is an instance of C, then this is known.

Epistemic interpretations

Assumptions

1. there is a fixed countably infinite set Δ that is the domain of every interpretation (Common Domain Assumption);
2. there is a mapping γ from the individuals to the domain elements that fixes the way individuals are interpreted (Rigid Term Assumption).

An epistemic interpretation is a pair (I,W), where

- I is a first-order interpretation and
- W is a set of first-order interpretations, all satisfying the above assumptions.

Interpretation function

For $\top$, $\bot$, for atomic concepts, negated atomic concepts, and for atomic roles, $\cdot^{I,W}$ agrees with $\cdot^{I}$.

- $(C \sqcap D)^{I,W} = C^{I,W} \cap D^{I,W}$
- $(\neg C)^{I,W} = \Delta \setminus C^{I,W}$
- $(\forall R.C)^{I,W} = \{a \in \Delta \mid \forall b.(a,b) \in R^{I,W} \rightarrow b \in C^{I,W}\}$
- $(\exists R.\top)^{I,W} = \{a \in \Delta \mid \exists b.\ (a,b) \in R^{I,W}\}$.

$$(\mathbf{K}C)^{I,W} = \bigcap_{J \in W} C^{J,W}.$$

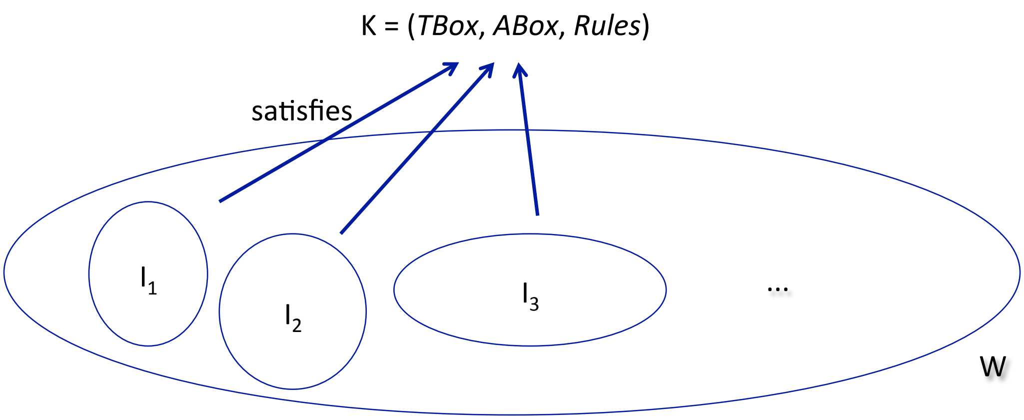
Epistemic model

for a rule knowledge base K = (*TBox*, *ABox*, *Rules*)

An epistemic model of K is a maximal nonempty set W = {I_1, I_2, …} such that

1. each I_j is a first-order interpretation,
2. for each I_j, the epistemic interpretation (I_j,W) satisfies the *TBox*, *ABox* and *Rules*.

Proposition. If (TBox, ABox) is first-order satisfiable then K has a unique epistemic model.



W may be seen as a maximal set of possible "worlds"

Each I_i defines an interpretation C^{Ii} of each class C and R^{Ii} of each role R of K.

(**K**C)^{I,W} is the set of objects that appear in each interpreation of C

Example

Let R consist of the rule

$$\mathbf{K}Student \sqsubseteq \forall eats.JunkFood$$

"those individuals that are known to be students eat only junk food".

K1 = (∅, A1, R),

A1 = {Student(PETER)}.

Let us determine the epistemic model W of K1.

The epistemic model W of K1

Every first-order interpretation I ∈ W must satisfy Student(PETER).

=> Peter is known to be a student.

$\mathbf{K}Student^{I,W} = \{ \gamma(PETER), \ldots \}$

W satisfies the $\mathbf{K}Student \sqsubseteq \forall eats.JunkFood$,

=> $\forall eats.JunkFood(PETER)$ holds in every I.

For any other domain element a ∈ Δ,

there is at least one interpretation in W where a is not a student.

=> Peter is the only domain element to which the rule applies.

The epistemic model of K1 consists exactly of the first order models of A1 ∪ {∀eats.JunkFood(PETER)}.

Example

K2 = (∅, A2, R), where
A2 = {¬∀eats.JunkFood(PETER)}.

¬∀eats.JunkFood(PETER) is true in every first-order interpretation of the epistemic model W.

maximality of W ⇒ there is at least one interpretation in W in which Peter is a student and another one where Peter is not a student.

therefore, Peter is not known to be a student. $\mathbf{K}Student^{I,W} \not\ni \gamma(PETER)$

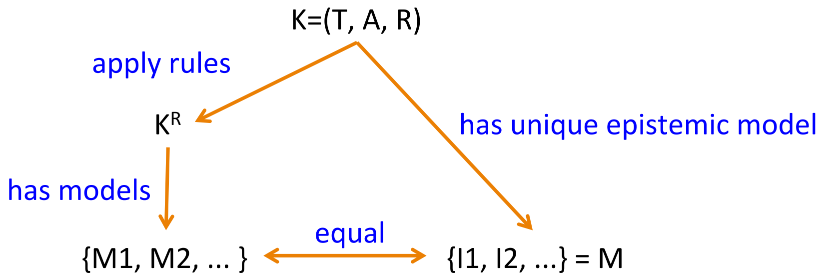
The epistemic model of K2 consists exactly of the first order models of A2.

The rule is satisfied because the antecedent is false.

Equivalence

The operational and epistemic interpretation coincide

Proposition. If M is the unique epistemic model of K = (*TBox*, *ABox*, *Rules*) then M consists of the first order models of K^{Rules} (obtained by rule application)



$\mathbf{K}C \sqsubseteq C$

Let W be an epistemic model of K = (T, A, R)

An epistemic model of K is a maximal nonempty set $W = \{I_1, I_2, ...\}$ such that for each I_j, the epistemic interpretation (I_j,W) satisfies the *TBox*, *ABox* and *Rules*.

$(\mathbf{K}C)^W = \bigcap_{J \in W} C^{J,W}$. so, for each I in W $(\mathbf{K}C)^W \subseteq C^{I,W}$

Let I be a member of W

$(\mathbf{K}C)^{I,W} = \bigcap_{J \in W} C^{J,W}. \subseteq C^{I,W}$ (because $I \in W$)

hence,

$\mathbf{K}C \sqsubseteq C$

holds in every member of W

$\mathbf{KK}C \sqsubseteq \mathbf{K}C$

Let W be an epistemic model of K = (T, A, R)

Let I be a member of W

$(\mathbf{KK}C)^{I,W} = \bigcap_{J \in W} (\mathbf{K}C)^{J,W} \subseteq (\mathbf{K}C)^{I,W}$ (because $I \in W$)

$\neg \mathbf{K}C \sqsubseteq \mathbf{K}\neg\mathbf{K}C$

Let W be an epistemic model of K = (T, A, R)

Let I be a member of W

$(\neg \mathbf{K}C)^{I,W} = \Delta \setminus (\mathbf{K}C)^{I,W}$

$= \Delta \setminus \bigcap_{J\in W} (\mathbf{K}C)^{J,W} = \bigcup_{J\in W} (\Delta \setminus (\mathbf{K}C)^{J,W})$

$= \bigcup_{J\in W} (\Delta \setminus \bigcap_{L\in W} C^{L,W}) = \bigcup_{J\in W} (\bigcup_{L\in W} \neg C^{L,W})$

$= \bigcup_{L\in W} \neg C^{L,W}$

$(\mathbf{K}\neg\mathbf{K}C)^{I,W} = \bigcap_{K\in W} (\neg \mathbf{K}C)^{K,W} = \bigcap_{K\in W} (\Delta \setminus (\mathbf{K}C)^{K,W})$

$= \bigcap_{K\in W} (\Delta \setminus (\bigcap_{L\in W} C^{L,W}) = \bigcap_{K\in W} (\bigcup_{L\in W} (\neg C)^{L,W})$

$= \bigcup_{L\in W} (\neg C)^{L,W}$